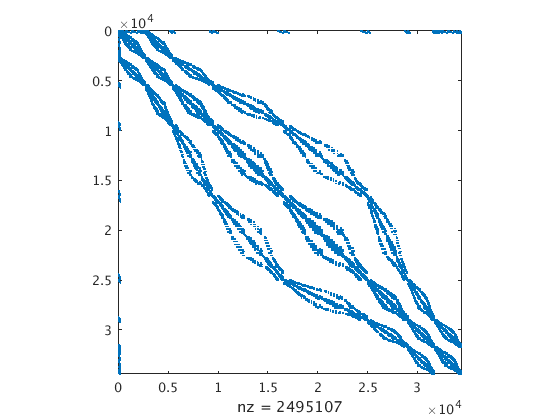
**Investigation on factorization preconditioners**

**(created in 04/18/2017)**

The investigated matrix is generated from abaqus software for SSI interaction model. The model includes two parts: 30X30X30m soil domain and a small 3-story structure, total dof is 34422.

After assembling the element matrices, Newmark effectivestiffness matrix is constructed to solve the linear system of equations.

*Sparsity pattern of the effectivestiffness matrix*

The condition number of effectivestiffness matrix(positive definite) is 1e13(which is very bad, usually around 1e6, 1e8) determines the convergence rate of the problem and also the number of accurate digits can be obtained( Timothy Sauer's numerical book).

Both incomplete LU and incomplete cholesky factorization are investigated inside the preconditioned conjuagate gradient method in Matlab.

ILU in matlab (recommended for BICG, GMRES):

5 options:

(1) type: no fill, crout, ilutp

(2) droptol:

(3) milu: off, row, column

(4) udiag: 0, 1

(5) thresh

|  |  |
| --- | --- |
| No fill | 1.8G, can not converge at 500 iterations |
| No fill, milu(row) | ill conditioned preconditioner |
| No fill, milu(col) | Most run to 1000 iterations but stagnated |
| Crout, droptol 1e-3 | 4G, pcg converge at iteration around 100, some stagnated |
| Crout, droptol 1e-4 | 10G, pcg converge at iteration around 50, some stagnated |
| Crout, droptol 1e-5 | 22G, pcg converge at iteration around 50, some stagnated |
| Crout, droptol 1e-3, milu(row) | 5G, pcg can not converge in 5000 iterations |
| Crout, droptol 1e-3, milu(column) | 5G, pcg can not converge in 5000 iterations |

ICHOL in matlab (recommended for PCG, MINRES)

5 options:

(1) type: no fill, crout, ilutp

(2) droptol:

(3) milu: off, row, column

(4) udiag: 0, 1

(5) thresh

|  |  |
| --- | --- |
| No fill | Non-positive pivot |
| No fill, michol on | Non-positvie pivot |
| No fill, diagcomp, alpha | Can not converge at 5000th iterations |
| ICT, droptol 1e-2 | Non-positive pivot |
| ICT, droptol 1e-3 | 1.8G, most converges at 50 iterations, some stagnated at 50th |
| ICT, droptol 1e-4 | 8G, most converges at 50 iterations, some stagnated at 50th |
| ICT, droptol 1e-5 | 14G, most converges at 50 iterations, some stagnated at 50th |
| ICT, droptol 1e-6 | 28G, most converges at 50 iterations, some stagnated at 50th |
| ICT, droptol 1e-3, diagcomp, alpha | 1.8G, cannot converge in 5000 iterations |
| ICT, droptol 1e-6, michol on | Nonpositive pivot |

Conclusion:

By trying ILU and ICHOL preconditioners in matlab with all possible options, it is found that incomplete cholesky factorization with dropping tolerance is the best possible preconditioner.

ILU is not suitable for positive difinite symmetric system while ICHOL is extremly suitable for finite element matrix, it is also written in KJ Bathe's book.